

Week 3: Lecture A

Improved Cipher Designs

Tuesday, September 2, 2025

Announcements

- **Project 1: Crypto** released (see [Assignments](#) page on course website)
 - **Deadline:** Thursday, September 18th by 11:59PM

Project 1: Cryptography

Deadline: Thursday, September 18 by 11:59PM.

Before you start, review the [course syllabus](#) for the Lateness, Collaboration, and Ethical Use policies.

You may optionally work alone, or in teams of **at most two** and submit **one project per team**. If you have difficulties forming a team, post on [Piazza's Search for Teammates](#) forum. Note that the final exam will cover project material, so you and your partner should collaborate on each part.

The code and other answers your group submits must be entirely your own work, and you are bound by the University's Student Code. You may consult with other students about the conceptualization of the project and the meaning of the questions, but you may not look at any part of someone else's solution or collaborate with anyone outside your group. You may consult published references, provided that you appropriately cite them (e.g., in your code comments). **Don't risk your grade and degree by cheating!**

Complete your work in the **CS 4440 VM**—we will use this same environment for grading. You may not use any **external dependencies**. Use only default Python 3 libraries and/or modules we provide you.

Helpful Resources

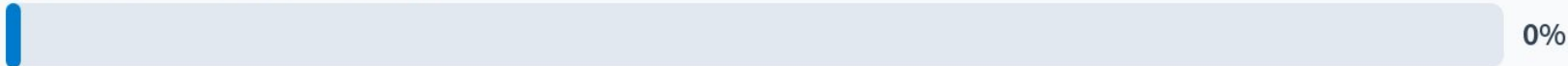
- [The CS 4440 Course Wiki](#)
- [VM Setup and Troubleshooting](#)
- [Terminal Cheat Sheet](#)
- [Python 3 Cheat Sheet](#)
- [PyMD5 Module Documentation](#)
- [PyRoots Module Documentation](#)

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Progress on Project 1

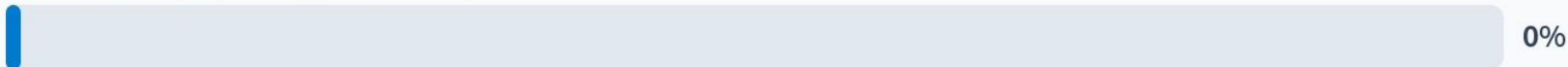
Finished both Part 1 and Part 2



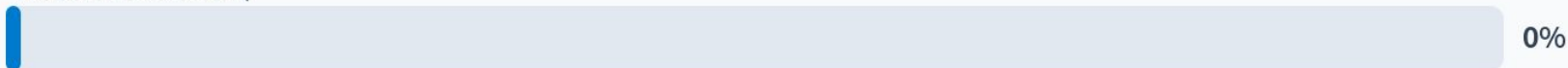
Finished only Part 1



Started but haven't finished Part 1



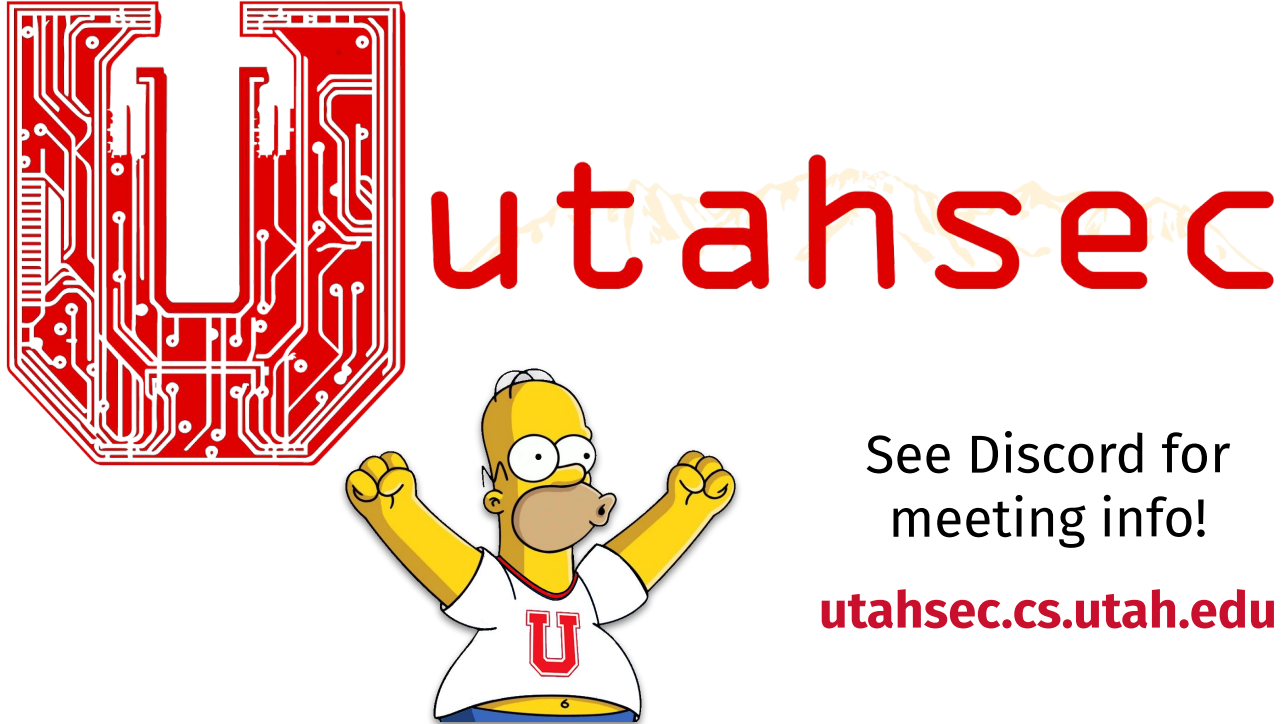
Haven't started :(



Project Tips

- Projects are challenging—**you're performing real-world attacks!**
 - Build off of lecture concepts
 - Make sure you understand the lectures
 - Prepare you to defend **in the real world**
- **Suggested strategy: get high-level idea down, then start implementing**
 1. Go through assignment and start sketching-out your approach
 2. **Come to Office Hours and ask if you're on the right track!**
 3. Then start building your program
- Don't get discouraged—**we are here to help!**
 - Most issues are cleared up in a few minutes of white-boarding

Announcements



See Discord for
meeting info!

utahsec.cs.utah.edu

Announcements



AI ACCELERATORS 101

With Daniel Kroening
Senior Principal Applied Scientist, Amazon



Kahlert

DISTINGUISHED COLLOQUIUM

Biography

Daniel Kroening is a Senior Principal Applied Scientist at Amazon, where he works on the correctness of the Neuron Compiler for distributed training and inference. Prior to joining Amazon, he worked as a Professor of Computer Science at the University of Oxford and is the co-founder of Diffblue Ltd., a University spinout that develops AI that targets code and code-like artefacts. He wrote the CBMC (for C), J BMC (for Java) and EBMC (for SystemVerilog) model checkers; CBMC is the engine of Kani (for verifying unsafe Rust). He has received the Semiconductor Research Corporation (SRC) Inventor Recognition Award, an IBM Faculty Award, a Microsoft Research SEIF Award, the Wolfson Research Merit Award, and the Rance Cleaveland Test-of-Time tool award. He serves on the CAV steering committee and was co-chair of FLOC 2018, EIC of Springer FMSD, and is co-author of the textbooks on Decision Procedures and Model Checking.



KAHLERT SCHOOL OF COMPUTING
THE UNIVERSITY OF UTAH

Abstract

LLMs are arguably among the largest technology investments since the moon landing, and rely on custom hardware accelerators both for training and inference. The talk will cover accelerating LLM transformer architectures using the combination of a compiler and a systolic compute array. The key enabler to achieving meaningful performance using the systolic compute array are deep program analyses of the model architecture in the Neuron Compiler. I will briefly report on our effort to build a verified (using Lean) compiler from XLA/HLO to the Trainium ISA.

**Thursday, September 11,
2025**

Kennecott Mechanical Engineering Building (MEK)

Room 3550

5:15 PM Speaker

6:15 PM Pizza



Questions?



Last time on CS 4440...

Message Confidentiality
Substitution Ciphers
Frequency Cryptanalysis

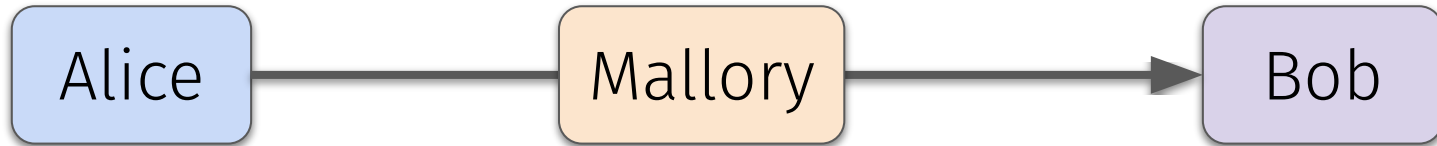
Message Confidentiality

- Confidentiality: ???



Message Confidentiality

- **Confidentiality:** ensure that only **trusted parties** can read the message
- Terminology: ???



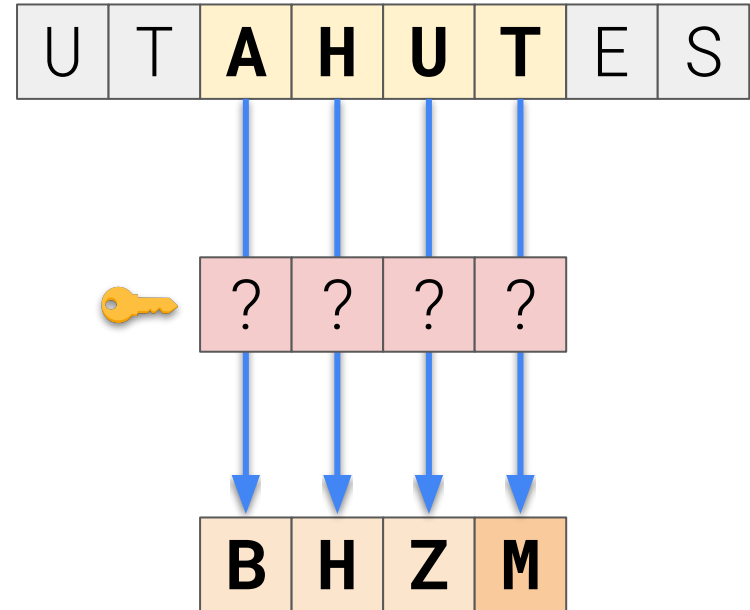
Message Confidentiality

- **Confidentiality:** ensure that only **trusted parties** can read the message
- Terminology:
 - **p** plaintext: original, readable message
 - **c** ciphertext: transmitted, unreadable message
 - **k** secret key: known only to Alice and Bob; facilitates $p \rightarrow c$ and $c \rightarrow p$
 - **E** encryption function: $E(p, k) \rightarrow c$
 - **D** decryption function: $D(c, k) \rightarrow p$



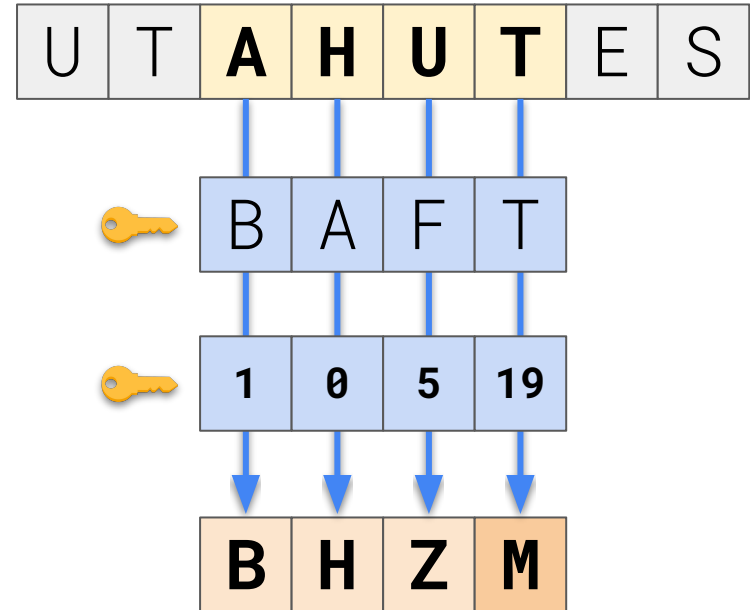
Confidentiality via Ciphers

- We define a key as ????



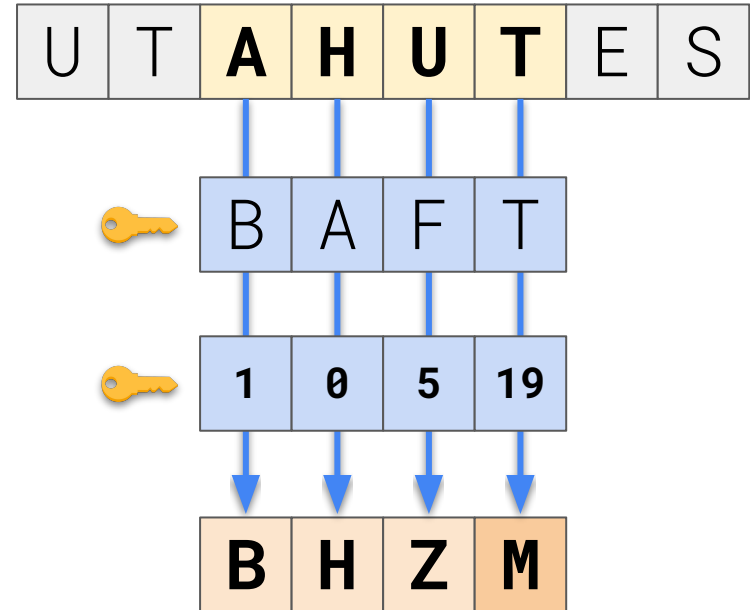
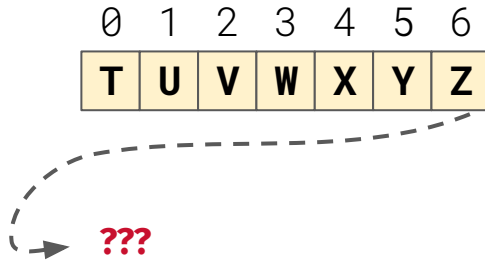
Confidentiality via Ciphers

- We define a key as a set of **shifts**
- Each shift represented by a **letter**
 - Relative position in the alphabet



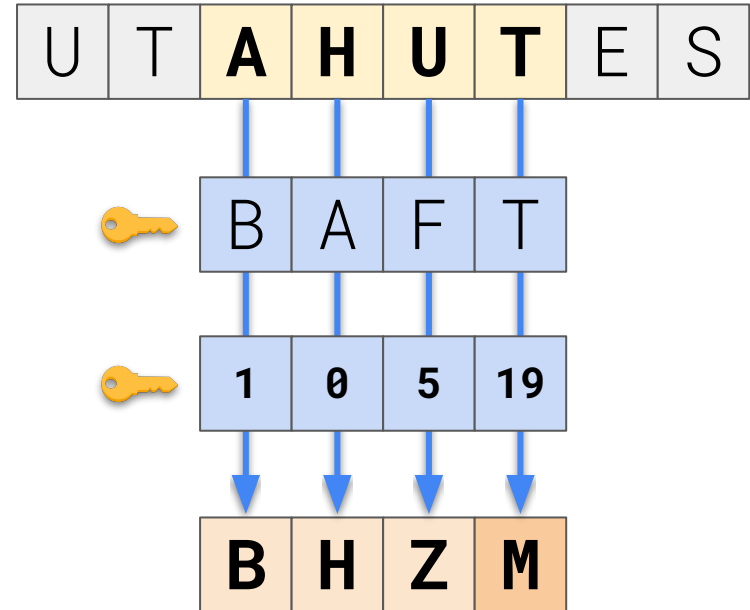
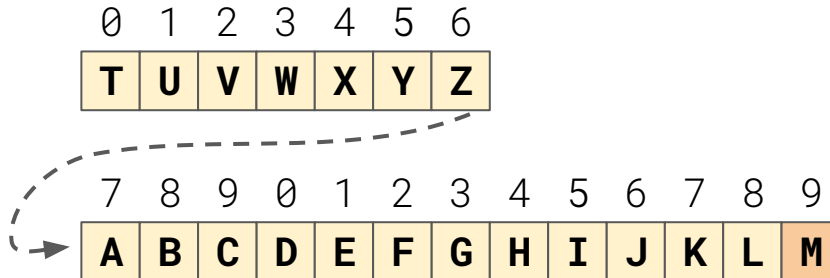
Confidentiality via Ciphers

- We define a key as a set of **shifts**
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 - Relative position in the alphabet
- Shift goes past end of alphabet?



Confidentiality via Ciphers

- We define a key as a set of **shifts**
- Each shift represented by a **letter**
 - Relative position in the alphabet
- Shift goes past end of alphabet?
 - **Wrap around** to beginning!



Caesar Ciphers

- Really old school cryptography
 - First recorded use: Julius Caesar (100–144 B.C.)
- Replaces each plaintext letter with ???



Caesar Ciphers

- Really old school cryptography
 - First recorded use: Julius Caesar (100–144 B.C.)
- Replaces each plaintext letter with one a fixed number of places down the alphabet
 - Encryption: $c_i := (p_i + k) \bmod 26$
 - Decryption: $p_i := (c_i - k) \bmod 26$
- Example for $k = 3$:
 - Plain: **ABCDEFGHIJKLMNOPQRSTUVWXYZ**
 - +Shift: **333333333333333333333333333333**
 - =Cipher: **DEFGHIJKLMNOPQRSTUVWXYZABC**

 - Plain: **go utes beat wash st**
 - +Key: **33 3333 3333 3333 33**
 - =Cipher: **jr xwhv ehdw zdvk vw**



Caesar Cipher Cryptanalysis



Brute-forcing
every possible key



Cryptanalysis

Caesar Cryptanalysis via Chi-Square Test

Example ciphertext string (with a **zero reverse shift**): LJSGUKJYSEKDLJGGAKWOGLHWLJNWFZLVEX

Expected English language letter frequencies:

```
{ "A": .08167, "B": .01492, "C": .02782, "D": .04253, "E": .12702, "F": .02228,  
  "G": .02015, "H": .06094, "I": .06966, "J": .00153, "K": .00772, "L": .04025,  
  "M": .02406, "N": .06749, "O": .07507, "P": .01929, "Q": .00095, "R": .05987,  
  "S": .06327, "T": .09056, "U": .02758, "V": .00978, "W": .02360, "X": .00150,  
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Caesar Cryptanalysis via Chi-Square Test

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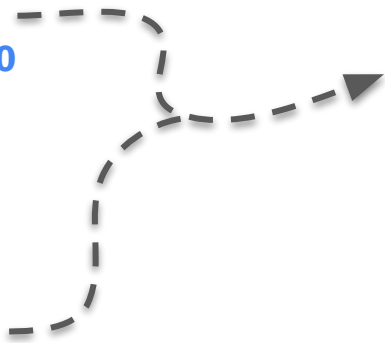
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```

$$\chi^2 = \sum_{i=1}^N \frac{(O_i - E_i)^2}{E_i}$$

$$\chi^2_L = (5.0 - 1.3685)^2 / 1.3685 \\ = 9.6367$$

O_L = observed count for letter 'L' = **5.0**

E_L = expected count for letter 'L'
= **EnglishFreq_L** * **StringLength**
= **0.04025** * **34**
= **1.3685**



Caesar Cryptanalysis via Chi-Square Test

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E_L = expected count for letter 'L'
= $\text{EnglishFreq}_L * \text{StringLength}$
= $0.04025 * 34$
= 1.3685

1. Add X^2 scores for all 26 alphabet letters
2. Final sum = **that reverse shift's X^2 score**
3. Repeat for the 25 other reverse shifts
4. **Lowest score** = the correct reverse shift
5. Mapped as forward shift = the key letter

Vigènere Ciphers

- First described by Bellaso in 1553
 - Later misattributed to Vigènere
- Encrypts successive letters via ???



Vigènere Ciphers

- First described by Bellaso in 1553
 - Later misattributed to Vigènere
- Encrypts successive letters via **sequence of Caesar ciphers** determined by the letters of a keyword
- For an **n**-letter keyword **k** ...
 - Encryption: $c_i := (p_i + k_{i \bmod n}) \bmod 26$
 - Decryption: $p_i := (c_i - k_{i \bmod n}) \bmod 26$
- Example for $k = \text{ABC}$ (i.e., $k_0 = 0, k_1 = 1, k_2 = 2$)
 - Plain: **bbbbbb amazon**
 - +Key: **012012 012012**
 - =Cipher: **bcdbcd anczpp**



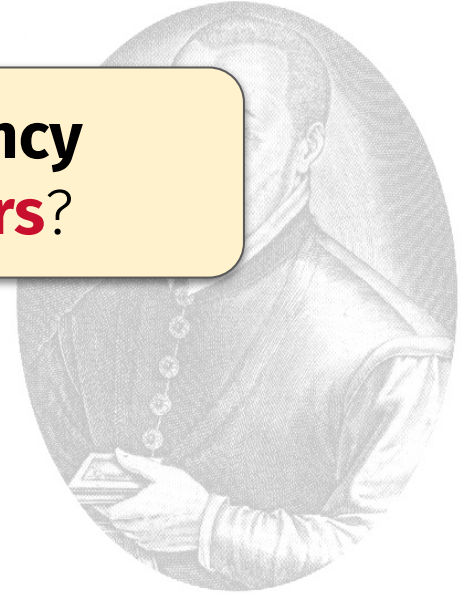
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Can we still perform **frequency analysis** for **Vigènere ciphers**?

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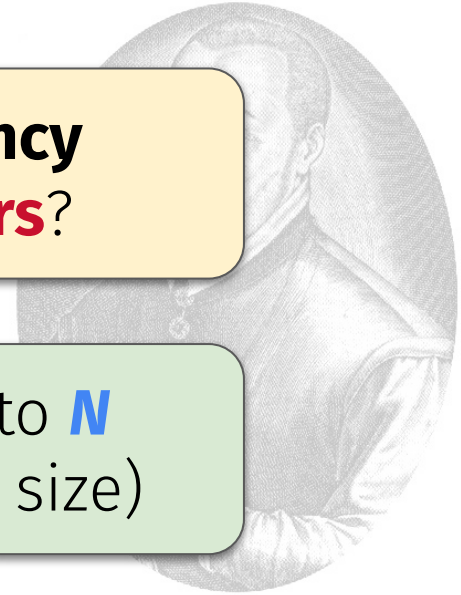
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Can we still perform **frequency analysis** for **Vigènere ciphers**?

- For an n -letter keyword k ...
 - Encryption: $c = (p + k) \bmod 26$
 - Decryption: $p = (c - k) \bmod 26$

- Example:
 - Plain: `anczpp`
 - +Key: `012012 012012`
 - =Cipher: `bcdbcd anczpp`

Yes—just partition it down into **N Caesar ciphers** (where **N** = key size)



Finding Key Size via Kasiski Method

■ Example:

p	THE	ARE	TW	OWAYS	OFCON	STRUC	TINGA	SOFTW	ARE	DE	SIGNO	NEWAY
	SYSTE	MSYST	EMSYS	TEMSY	STEMS	YSTEM	SYSTE	MSYST	EMSYS	TEMSY		
c	LFWKI	MJCLP	SISWK	HJOGL	KMVGU	RAGKM	KMXMA	MJCVX	WUYLG	GI	ISW	
p	ISTOM	AKEIT	SOSIM	PLETH	ATTHE	REARE	OBVIO	USLYN	ODEFI	CIENC		
	STEMS	YSTEM	SYSTE	MSYST	EMSYS	TEMSY	STEMS	YSTEM	SYSTE	MSYST		
c	ALXAE	YCXMF	KMKBQ	BDCLA	EFLFW	KIMJC	GUZUG	SKECZ	GBWYM	OACFV		
p	IESAN	DTHEO	THERW	AYIST	OMAKE	ITSOC	OMPLI	CATED	THATT	HEREA		
	EMSYS	TEMSY	STEMS	YSTEM	SYSTE	MSYST	EMSYS	TEMSY	STEMS	YSTEM		
c	MQKYF	WXTWM	LAIDO	YQBWF	GKSDI	ULQGV	SYHJA	VEFWB	LAEFL	FWKIM		
p	RENOO	BVIOU	SDEFI	CIENC	IESTH	EFIRS	TMETH	ODISF	ARMOR	EDIFF		
	SYSTE	MSYST	EMSYS	TEMSY	STEMS	YSTEM	SYSTE	MSYST	EMSYS	TEMSY		
c	JCFHS	NNGGN	WPWDA	VMQFA	AXWFZ	CXBVE	LKWML	AVGKY	EDEMJ	XHUXD		

Finding Key Size via Kasiski Method

- Pick **realistic key lengths**; a length of two or three is probably short

Dist.	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
74	x																		
72	x	x	x		x		x	x									x		
66	x	x			x					x									
36	x	x	x		x			x									x		
32	x		x				x								x				
30	x	x		x	x				x					x					

Finding Key Size via Kasiski Method

- Then, **group letters by columns**—they received equal shifts!

123456 LFWKIM	123456 JCLPSI	123456 SWKHJO	123456 GLKMVG	123456 URAGKM	123456 KMXMAM	123456 JCVXWU	123456 YLGII	123456 SWALXA
123456 EYCXMF	123456 KMKBQB	123456 DCLAEF	123456 LFWKIM	123456 JCGUZU	123456 GSKECZ	123456 GBWYMO	123456 ACFVMQ	123456 KYFWXT
123456 WMLAID	123456 OYQBWF	123456 GKSDIU	123456 LQGVSY	123456 HJAVEF	123456 WBLAEF	123456 LFWKIM	123456 JCFHSN	123456 NGGNWP
123456 WDAVMQ	123456 FAAXWF	123456 ZCXBVE	123456 LKWMLA	123456 VGKYED	123456 EMJXHU	12 XD		

Recap: Breaking Vigènere

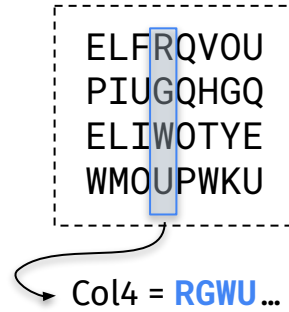
1. ???

Recap: Breaking Vigènere

1. Identify the key length:
 - **Project 1:** keys will always be of length **eight**
 - **Extra Credit:** key varies—use **Kasiski method**!
2. ???

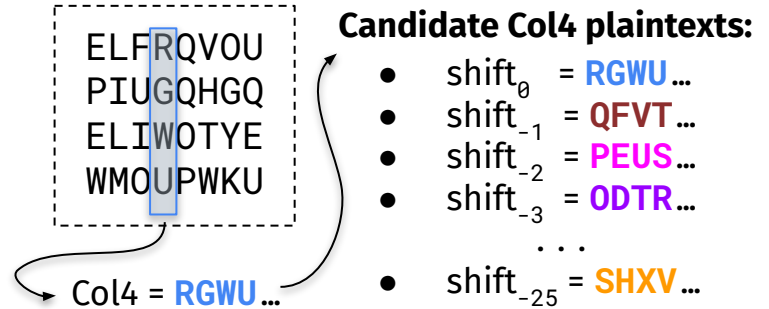
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1. Identify the key length:
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2. Divide ciphertext into **N** columns:
 - **Why?** Because Vigènere uses a repeating key
 - Vigènere cipher is a set of **N** Caesar ciphers
3. ???



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3. Perform cryptanalysis on each column:
 - Find all candidate **reverse shifts** per column



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 - Find all candidate **reverse shifts** per column
 - **Chi-square test:** find **best-fit reverse shift**

ELFRQVOU
PIUGQHGQ
ELIWOTYE
WMOUPWKU

Col4 = **RGWU**...

Candidate Col4 plaintexts:

- shift₀ = **RGWU**...
- shift₋₁ = **QFVT**...
- shift₋₂ = **PEUS**...
- shift₋₃ = **ODTR**...
- ...
- shift₋₂₅ = **SHXV**...

Candidate Col4 X² scores:

- shift₀ = **10.50**
- shift₋₁ = **20.02**
- shift₋₂ = **5.135**
- shift₋₃ = **2.156**
- ...
- shift₋₂₅ = **13.31**

$$X^2 = X_A^2 + X_B^2 + X_C^2 + \dots + X_Z^2$$

Recap: Breaking Vigènere

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 - Compute forward shift = **column's key letter**

Candidate Col4 plaintexts:

- $\text{shift}_0 = \text{RGWU} \dots$
- $\text{shift}_{-1} = \text{QFVT} \dots$
- $\text{shift}_{-2} = \text{PEUS} \dots$
- $\text{shift}_{-3} = \text{ODTR} \dots$
- $\dots$
- $\text{shift}_{-25} = \text{SHXV} \dots$

Col4 = **RGWU**...

Candidate Col4 X^2 scores:

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$$X^2 = X_A^2 + X_B^2 + X_C^2 + \dots + X_Z^2$$

Smallest X^2 = correct reverse shift for Col4!

Recap: Breaking Vigènere

1. Identify the key length:
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3. Perform cryptanalysis on each column:
 - Find all candidate **reverse shifts** per column
 - **Chi-square test:** find **best-fit reverse shift**
 - Compute forward shift = **column's key letter**
 - Assemble all **N** column keys = **the Vigènere key**!

ELFRQVOU
PIUGQHGQ
ELIWOTYE
WMOLPWKL

Candidate Col4 plaintexts:

- $\text{shift}_0 = \text{RGWU} \dots$
- $\text{shift}_{-1} = \text{QFVT} \dots$
- $\text{shift}_{-2} = \text{PEUS} \dots$
- $\text{shift}_{-3} = \text{ODTR} \dots$

Rinse and repeat for remaining columns Col1, Col2, Col3, ... !

- $\text{shift}_0 = 10.50$
- $\text{shift}_{-1} = 20.02$
- $\text{shift}_{-2} = 5.135$
- $\text{shift}_{-3} = 2.156$
- ...
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Questions?



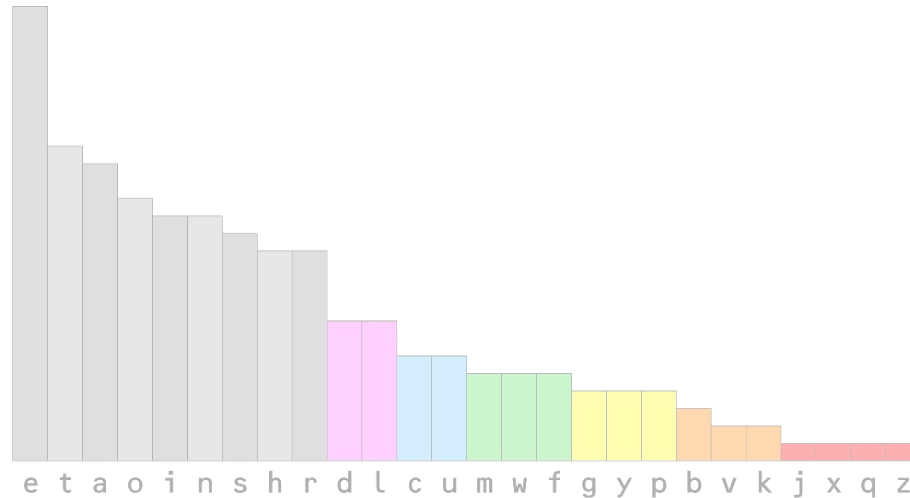
This time on CS 4440...

Pseudo-random Keys
One-time Pads
Transposition Ciphers
Cipher Metrics

Pseudo-random Keys

Recap: Confidentiality via Substitution Ciphers

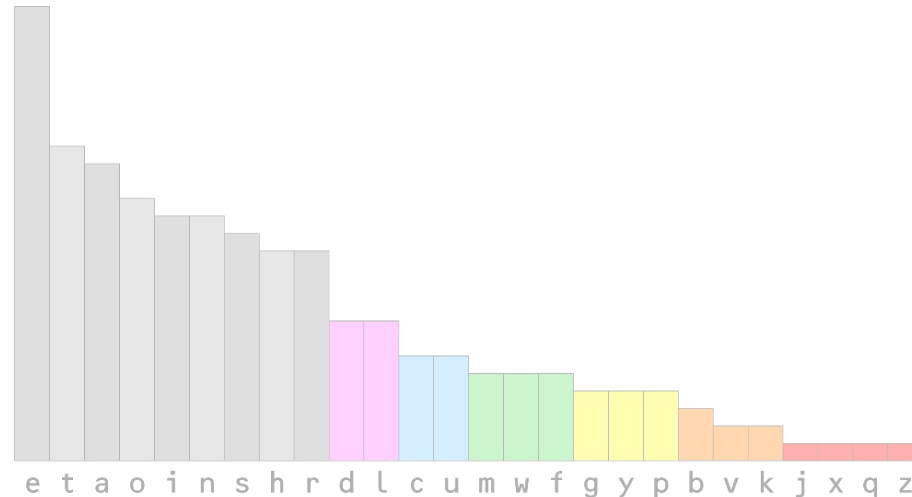
- Clearly, **simple substitution ciphers** are vulnerable to frequency analysis
 - Root cause: ???



Ordered by frequency

Recap: Confidentiality via Substitution Ciphers

- Clearly, **simple substitution ciphers** are vulnerable to frequency analysis
 - Root cause:** the key length is **much smaller** than the plaintext length



Ordered by frequency

Recap: Confidentiality via Substitution Ciphers

- Clearly, **simple substitution ciphers** are vulnerable to frequency analysis
 - Root cause:** the key length is **much smaller** than the plaintext length

How can we create **a better key**
to improve **confidentiality**?



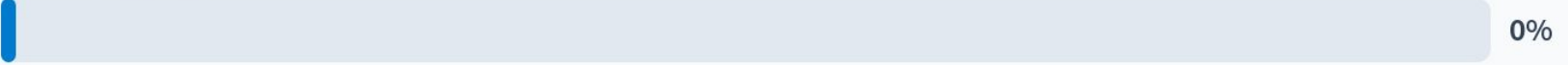
Ordered by frequency

How long should an ideal cipher key be?

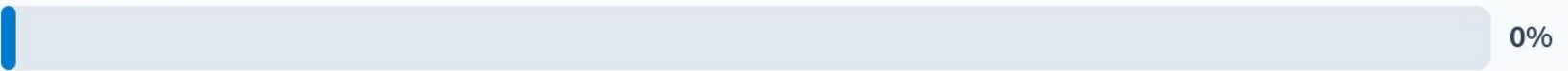
Half the size of the plaintext



As long as the plaintext



None of the above



Recap: Confidentiality via Substitution Ciphers

- Clearly, **simple substitution ciphers** are vulnerable to frequency analysis
 - Root cause:** the key length is **much smaller** than the plaintext length

How can we create **a better key**
to improve **confidentiality**?

Plaintext-length keys will deter
frequency analysis!



e t a o i n s h r d l c u m w f g y p b v k j x q z

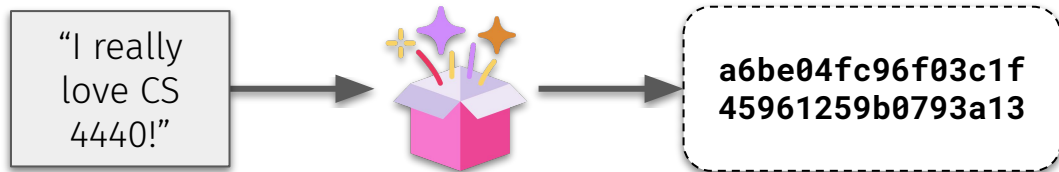
Ordered by frequency

Generating Keys

- **Functions: ???**

Generating Keys

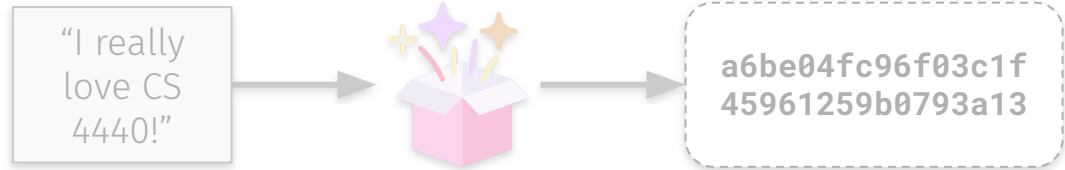
- **Functions:** takes input and generates output
 - E.g., Hash functions
 - E.g., HMAC functions



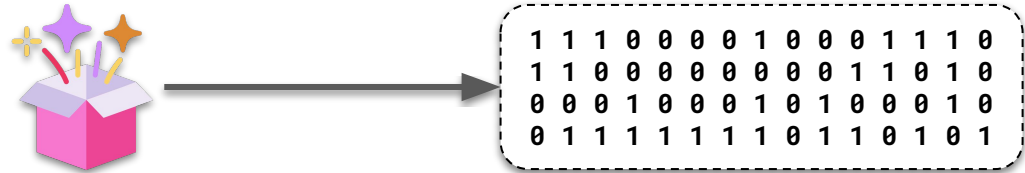
- **Generators: ???**

Generating Keys

- **Functions:** takes input and generates output
 - E.g., Hash functions
 - E.g., HMAC functions



- **Generators:** produces output out of thin air
 - E.g., number generators
 - E.g., HMAC secret keys



An ideal key is *random*...



What are some physical sources of randomness?

Nobody has responded yet.

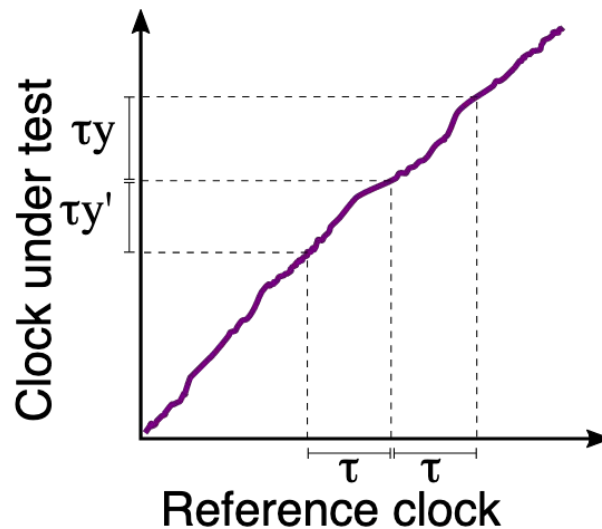
Hang tight! Responses are coming in.



Generating Random Keys

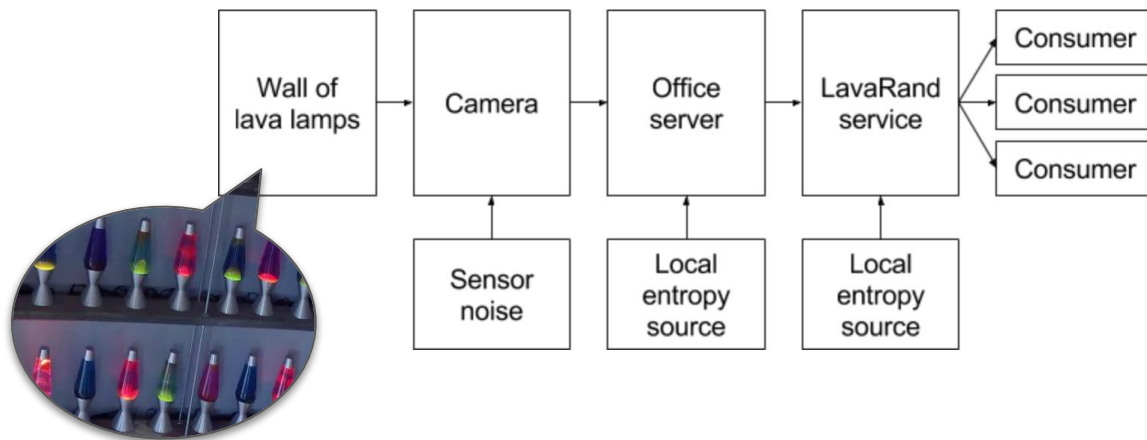
■ Physical randomness:

- Coin flips
- Atomic decay
- Thermal noise
- Electromagnetic noise
- Physical variation
 - Clock drift
 - DRAM decay
 - Image sensor errors
 - SRAM startup-state
- Lava Lamps



Generating Random Keys

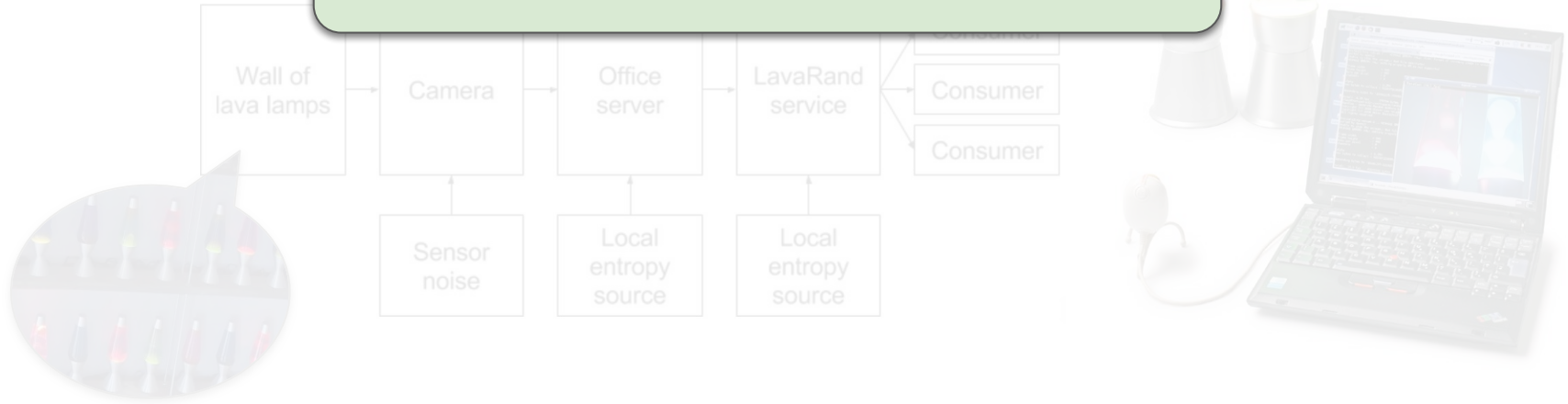
- Harnessing physical randomness: “LavaRand”
 - True randomness from lava lamps
 - Used by CloudFlare today



Generating Random Keys

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Highest guarantees of **security**

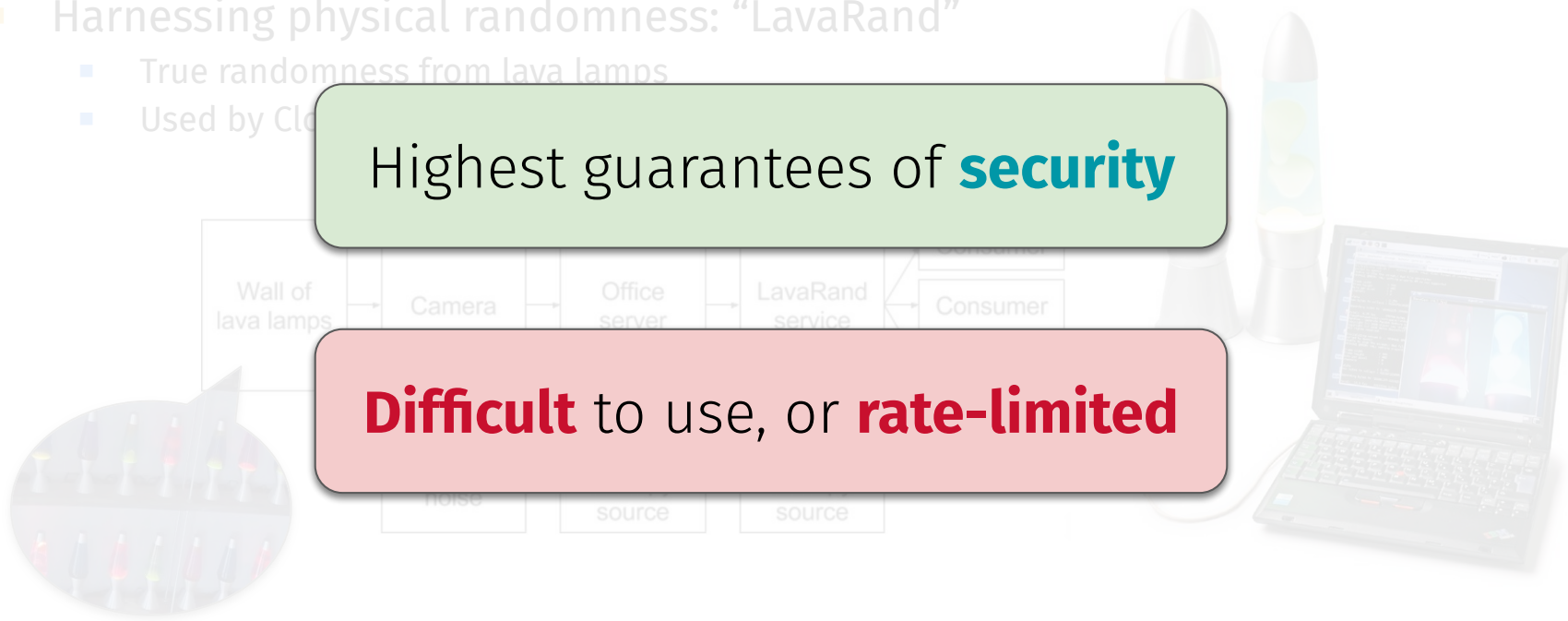


Generating Random Keys

- Harnessing physical randomness: “LavaRand”
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Highest guarantees of **security**

Difficult to use, or **rate-limited**



“Pseudo” Randomness

- What is **true randomness**?
 - **Physical** process that's inherently random
 - **Secure** yet **impractical**
 - Scarce, hard to use
 - Rate-limited



“Pseudo” Randomness

- What is **true randomness**?
 - **Physical** process that's inherently random
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 - Rate-limited
- Pseudo-random generator (PRG)
 - **Input:** a small **seed** that is **truly random**
 - **Output:** long sequence that **appears random**



“Pseudo” Randomness

- What is **true randomness**?
 - **Physical** process that's inherently random
 - **Secure** yet **impractical**
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PRGs offer the best of both worlds: **practical** (fast, easy-to-use) and **secure** (appear random)

- Pseudo-randomness
 - **Output:** long sequence that **appears random**



Pseudo-random Generators (PRGs)

- We say a **PRG** is **secure** if Mallory can't do better than random guessing

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Pseudo-random Generators (PRGs)

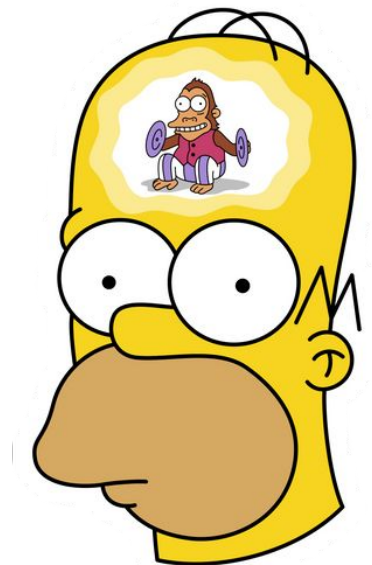
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Pseudo-random Generators (PRGs)

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- **Problem:** How much **true randomness** is **enough**?
 - **Example:** **one coin flip** = Mallory needs **very few tries** to guess
- **Problem:** Is our “true randomness” **truly random**?
 - **Example:** coin flip output = **one in two**. Lava lamps have way more!
- **Solutions:**
 - Generate a bunch of true randomness **over a long time** from a **high entropy source**
 - Run through a **PRF** to get an easy-to-work-with, **fixed-length** randomness (e.g., 256 bits)

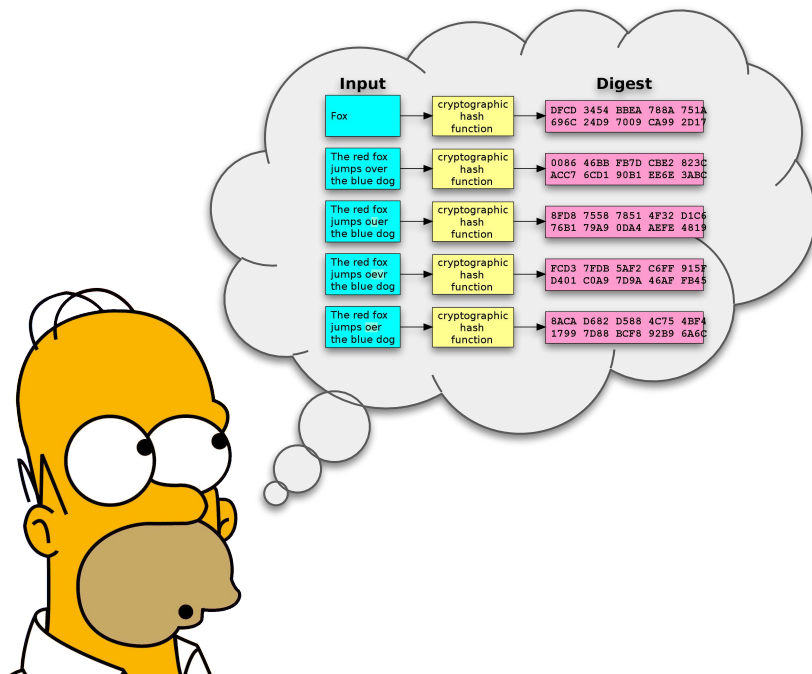
Constructing a PRG

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Constructing a PRG

- **Idea:** Build a **PRG** using a **PRF**
- **Observation:** **PRF**, given consecutive inputs, produce outputs that are randomly distributed (hopefully)
- **Result:** For **truly-random** s and **PRF** f :
 - **Pseudo-random generated string** =
 $f_s(0) \parallel f_s(1) \parallel f_s(2) \parallel f_s(3) \dots$



Proving a PRG is Secure

- **Theorem:** if f is a **secure PRF**
 - ... and g is seeded from f
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- **Proof:** if f is a **secure PRF**, we must show that g is a **secure PRG**
 1. Assume g actually is **insecure**... then Mallory can break it
 2. If that were true, Mallory could also break the **PRF** too
 3. This would **contradict** the fact that f is a **secure PRF**!

Proving a PRG is Secure

- **Theorem:** if f is a **secure PRF**

- ... and g is seeded from f
- ... then g is a **secure PRG**

How should we **seed** our PRG?

- **Proof:** if f is a **secure PRF** and g is a **PRG**

1. Assume g actually is **insecure**... then Mallory can break it
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What happens if we **fail**?

Proving a PRG is Secure

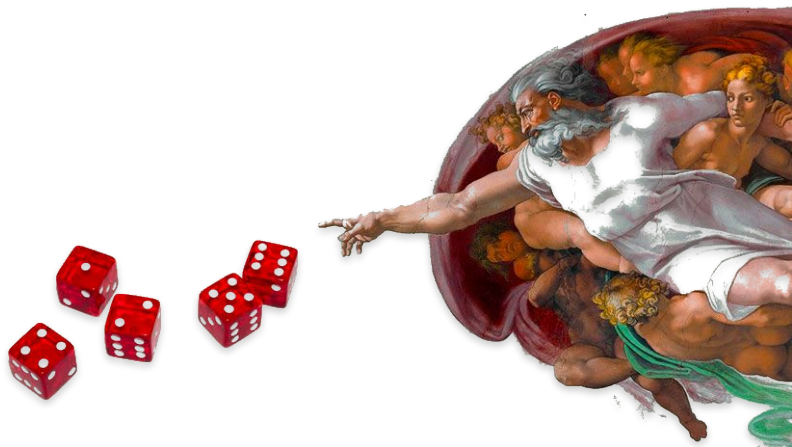
- **Theorem:** if f is a **secure PRF**
 - ... and g is seeded from f
 - ... the

- **Proof:** if f
 1. Assu
 2. If tha
 3. This v

When our assumptions hold, we transform a small amount of **“true” randomness** into a *wealth* of **“apparent” randomness**

Practical Randomness

- Where do you get **true** randomness?
- Modern OSes typically collect randomness
- They give you API calls to capture it
- e.g., Linux:
 - `/dev/random` is a device that gives random bits; it blocks until available
 - `/dev/urandom` gives output of a PRG; nonblocking; seeded from `/dev/random` eventually



Questions?



Plaintext-length Keys: One-time Pads

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- Alice and Bob generate a **plaintext-length** string of **random bits**: the one-time pad **k**

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A	B	Q
0	0	0
0	1	1
1	0	1
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$$a \text{ XOR } b \text{ XOR } b = a$$

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- To be secure:
 - Key must be **truly random**
 - Key must never be **reused**



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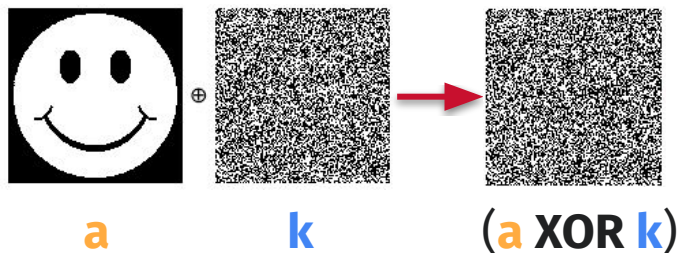
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Attacking OTPs: Non-random Keys

- Suppose the key bits **aren't** truly random
 - E.g., generated by selecting one of three values
- **How would this help Mallory?**

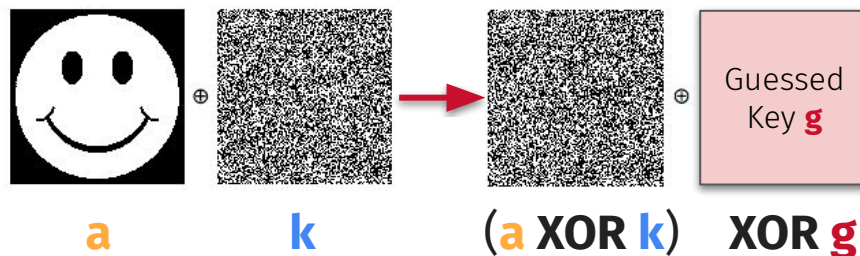
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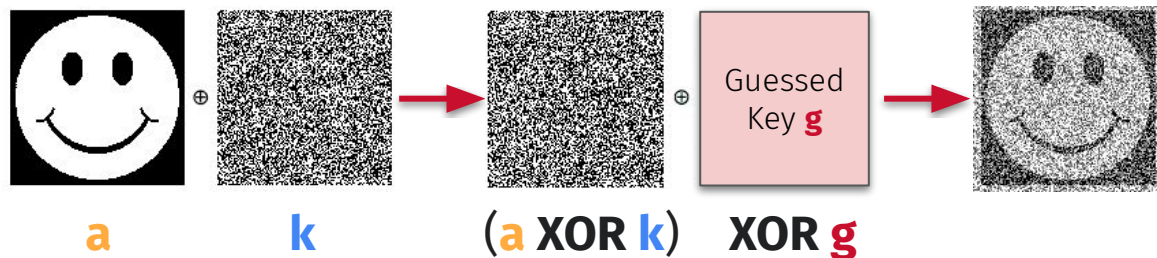
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- **How would this help Mallory?**
 1. She intercepts an encrypted message
 2. She **guesses key values** and decrypts
 3. She can **recover** parts of the plaintext!

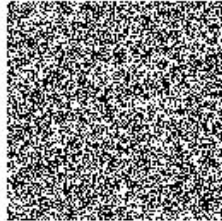


Attacking OTPs: Key Reuse

(a XOR k)

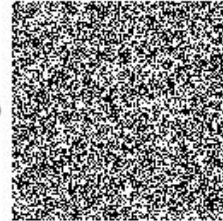


⊕



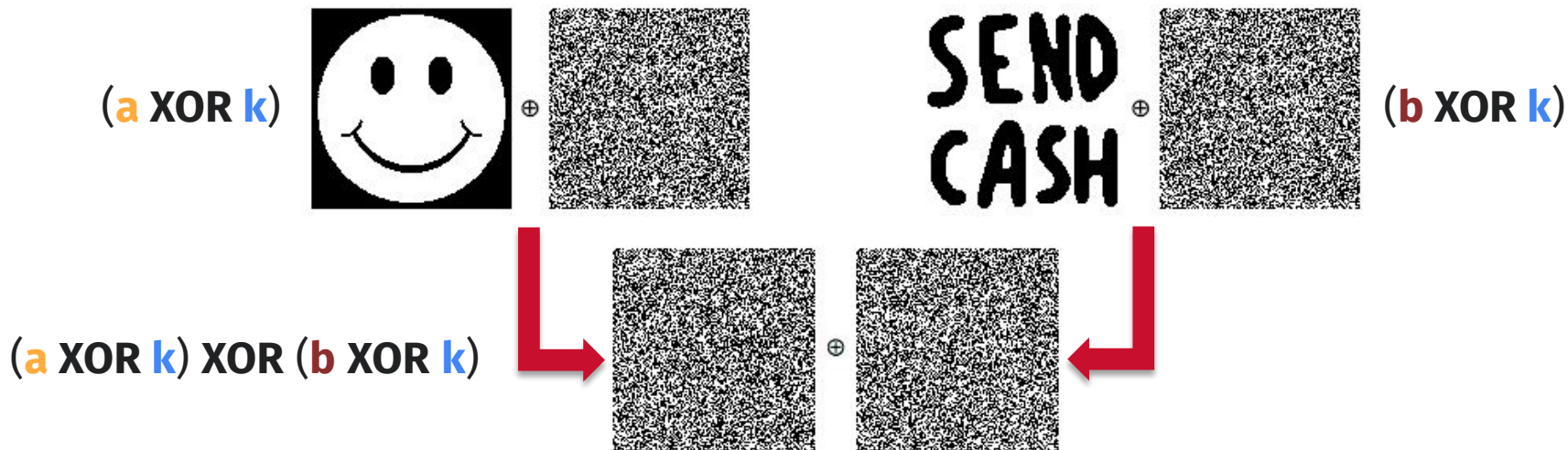
SEND
CASH

⊕

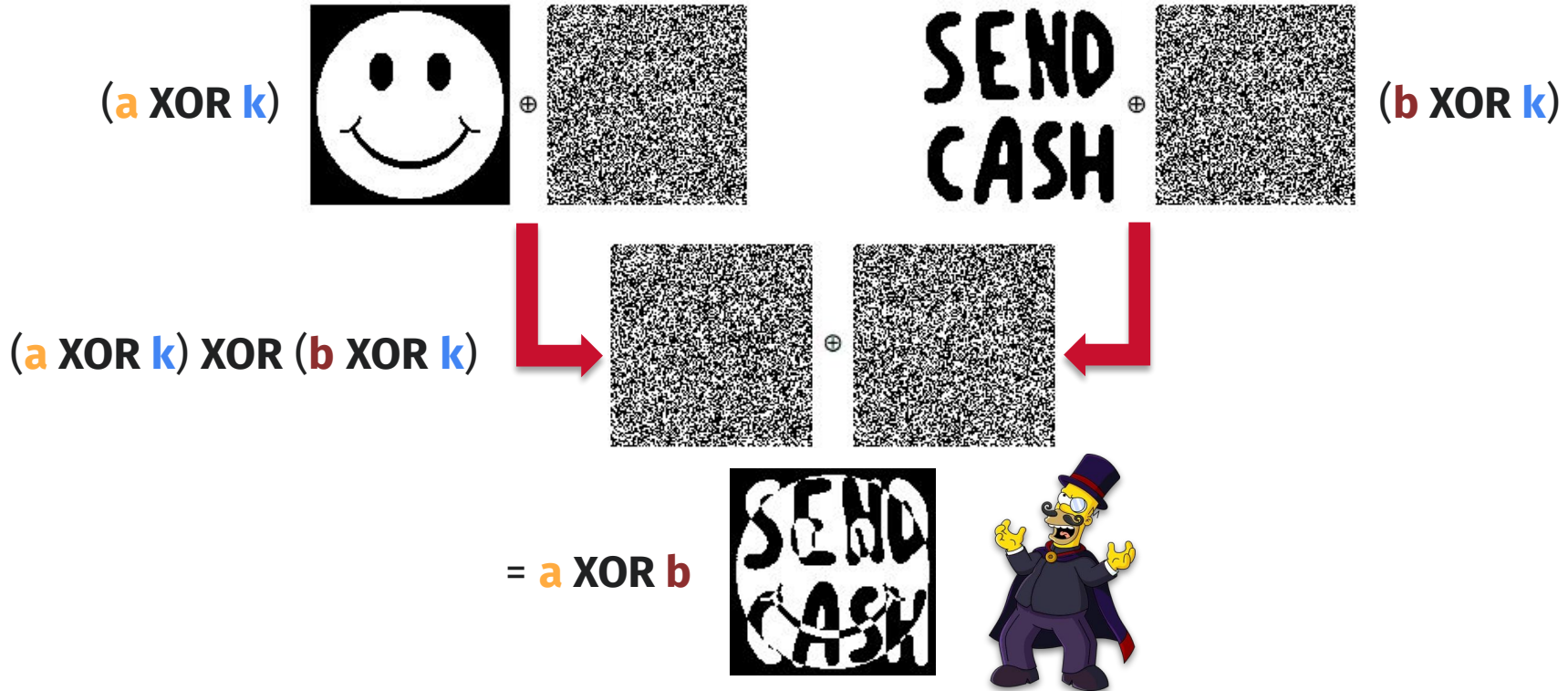


(b XOR k)

Attacking OTPs: Key Reuse



Attacking OTPs: Key Reuse



One-time Pads

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- Encryption:
- Decryption:

Provably **Secure**
(if key is **random** + not **reused**)

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- To be secure:

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Highly **Impractical**



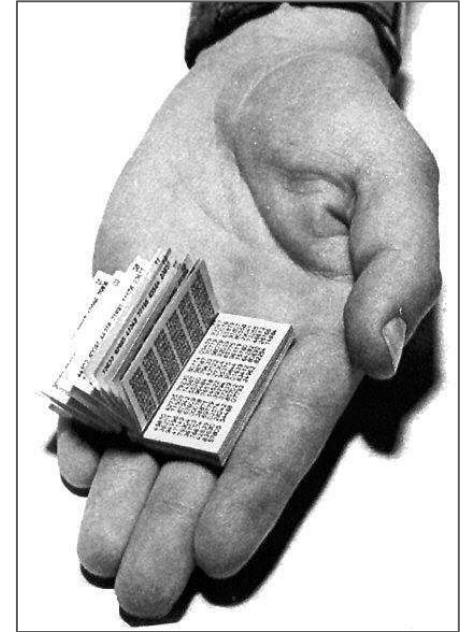
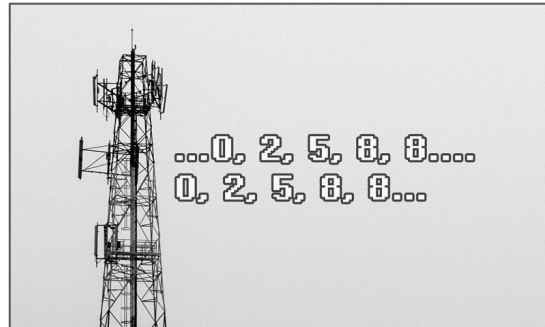
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Impracticality of OTPs

- Generating OTPs
 - Slow and/or rate-limited
 - By hand, LavaRand, etc.
- Deploying OTPs
 - Potentially very long
 - Challenging to conceal
- Cold War numbers stations
 - Encrypted message sent via short-wave radio to agents
 - Agent decrypts with their OTP
 - Throw OTP away after!
 - Many remain in service today!
 - Lincolnshire Poacher



Questions?



Plaintext-length Keys: Stream Ciphers

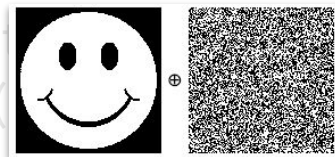
Stream Cipher

- **Idea:** Use a **Pseudo-random Generator** instead of a truly random pad
- **Recall:** a secure PRG inputs a **true-random seed**, outputs a stream that's **indistinguishable** from true randomness (unless attacker **knows seed**)
 1. Start with a shared secret **truly random seed** (from a lava lamp, mouse clicks, etc.)
 2. Alice & Bob each use this seed to seed their PRG and generate **k bits of PRG output**
 3. To encrypt and decrypt, perform the same operations as the One-time Pad:
 - Encryption: $c_i := p_i \text{ XOR } k_i$
 - Decryption: $p_i := c_i \text{ XOR } k_i$

Stream Cipher

- Idea: Use a **Pseudo-random Generator** instead of a truly random pad

What if you **reuse** the PRG's **random seed** or its **output**?



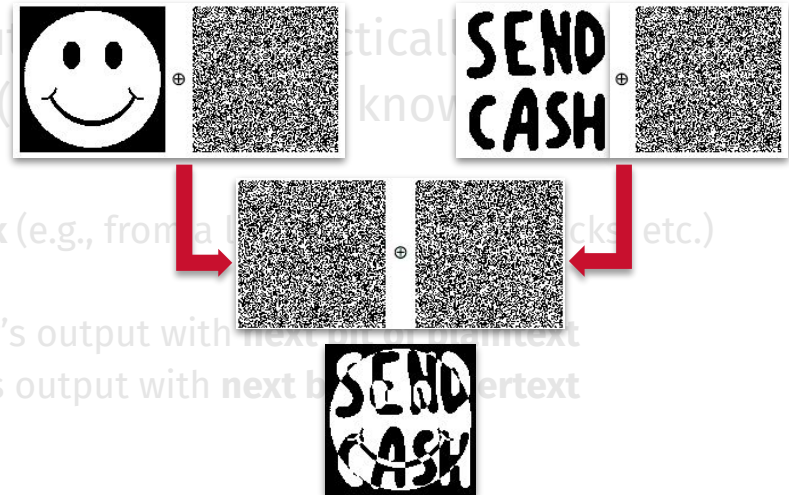
1. Start with shared secret **truly random** number **k** (e.g., from a lava lamp, mouse clicks, etc.)
2. Alice & Bob each use **k** to seed their PRG
3. To encrypt, **Alice XORs next bit** of her generator's output with **next bit of plaintext**
4. To decrypt, **Bob XORs next bit** of his generator's output with **next bit of ciphertext**

Stream Cipher

- Idea: Use a **Pseudo-random Generator** instead of a truly random pad

What if you **reuse** the PRG's **random seed** or its **output**?

Vulnerable to partial (or full) recovery of the **plaintext**!



Stream Cipher

- **Idea:** Use a pseudorandom generator instead of a truly random pad
- **Recall:** Secure PRG inputs a seed k , outputs a stream practically indistinguishable from random (Alice knows k)

What is the tradeoff between
an **OTP** and **Stream Cipher**?

1. Start with shared seed k
2. Alice & Bob each have a PRG
3. To encrypt, Alice XORs next bit of her generator's output with next bit of plaintext
4. To decrypt, Bob XORs next bit of his generator's output with next bit of ciphertext

Questions?



Transposition Ciphers

Transposition Ciphers

- **Substitution** ciphers swap-out plaintext symbols for others
 - E.g., shifting, XORing, etc.
- We've learned about several substitution ciphers
 - E.g., Caesar, Vigenere, one-time pad, stream cipher
- Can we come up with an alternative to **substitution**?

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- We've learned about several substitution ciphers
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- **Can we come up with an alternative to substitution?**
- **Transposition:** rearrange plaintext symbols to create ciphertext

Columnar Transposition

- Rearrange plaintext symbols to create ciphertext
 - Create a table with $|k|$ columns and $\lceil |p|/|k| \rceil$ rows (k is the keyword)
 - Place plaintext symbols in columns (left to right), cycling around to next row of the first column when current row of last column is filled
 - Create the ciphertext by writing entire columns (as a serial stream) to the output, where the keyword determines the column order

- Example:

6	3	2	4	1	5
---	---	---	---	---	---

- k = "ZEBRAS" (632415)
- p = "We are discovered flee at once"

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W	E	A	R	E	D
I	S	C	O	V	E
R	E	D	F	L	E
E	A	T	O	N	C
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 - $k = \text{"ZEBRAS"} (632415)$
 - $p = \text{"We are discovered flee at once"}$
 - $c = \begin{array}{lll} \text{EVLN} & \text{ACDT} & \text{ESEA} \\ \text{ROFO} & \text{DEEC} & \text{WIREE} \end{array}$

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- Example:
 - k = "ZEBRAS" (632415)
 - p = "We are discovered flee at once"
 - c = EVLNX ACDTQ ESEAM
ROFOP DEECD WIREE
 - Replace **null** with nonsense symbol

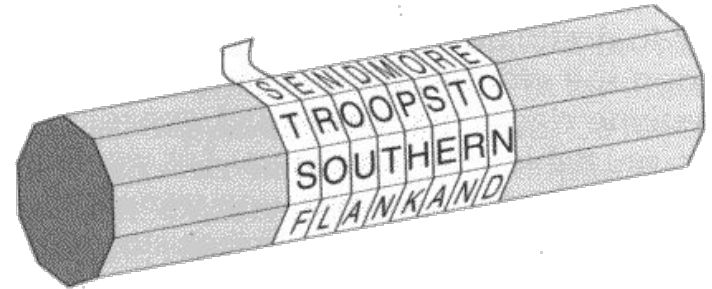
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Rail Fence (aka Zig Zag or Scytale) Cipher

- Rearrange plaintext on downwards, diagonally successive “rails”

W				E				C				R				L				T				E
	E		R		D		S		O		E		E		F		E		A		O		C	
		A				I				V				D				E				N		

- c** = WECRLTE ERDSOEEFEAO C AIVDEN

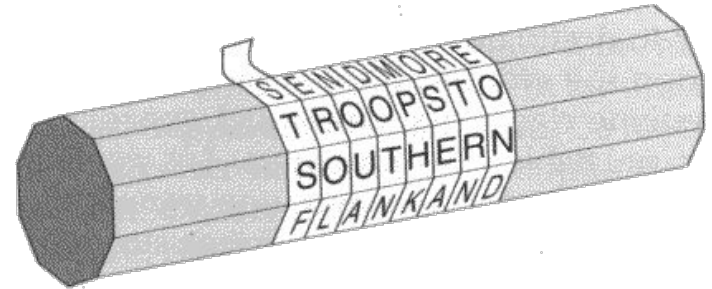


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	E		R		D		S		O		E		E		F		E		A		O		C	
		A				I				V				D				E				N		

- c** = WECRLTE ERDSOEEFEAOC AIVDEN
- Decryption:** use same-diameter cylinder!



Columnar Cipher Cryptanalysis

- What does a **brute force** attack look like?



Columnar Cipher Cryptanalysis

- What does a **brute force** attack look like?
 1. Guess number of columns
 2. Rearrange ciphertext in (probably) wrong order
 3. Look for anagrams to get correct order
 - Harder if null characters are rewritten
- Weakness of a transposition cipher?



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 - Harder if null characters are rewritten
- Weakness of a transposition cipher?
 - **Plaintext** characters end up in the ciphertext

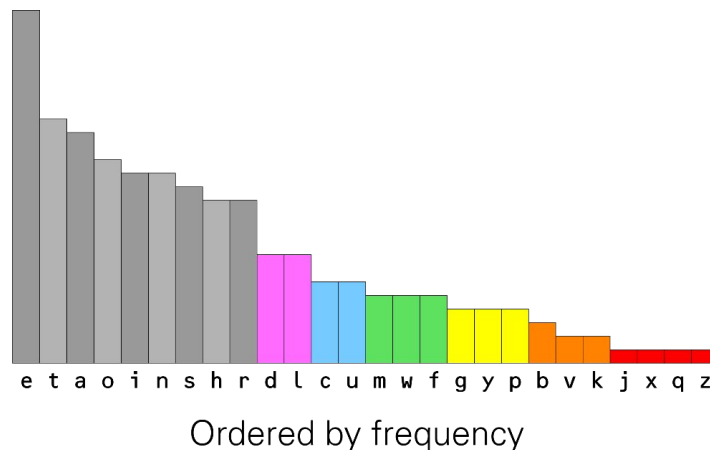


Is it transposition or substitution?

- Given a message ciphertext, how can you determine whether a transposition or a substitution cipher encrypted the plaintext?
 - **Hint:** frequency analysis

Is it transposition or substitution?

- Given a message ciphertext, how can you determine whether a transposition or a substitution cipher encrypted the plaintext?
 - **Hint:** frequency analysis
- **Transposition:**
 - Letters have **expected** letter frequencies
- **Substitution:**
 - Letters have **different** letter frequencies



Stronger Transposition

- How would you build a stronger columnar transposition cipher?
- **Transpose multiple times** with same or different keywords

6	3	2	4	1	5
W	E	A	R	E	D
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5	6	4	2	3	1
E	V	L	N	A	C
D	T	E	S	E	A
R	O	F	O	D	E
E	C	W	I	R	I
E	null	null	null	null	null

k_2 = "STRIPE" (632415)
 c_2 = CAEIX NSOIN AEDRX
 LEFWS EDREE VTOCG

Stronger Transposition

- How would you build a stronger columnar transposition cipher?
- **Transpose multiple times** with same or different keywords
 - **Myszkowski Transposition** on recurring letters in key

T	O	M	A	T	O
5	3	2	1	6	4
W	E	A	R	E	D
I	S	C	O	V	E
R	E	D	F	L	E
E	A	T	O	N	C
E	null	null	null	null	null

c = ROFOXACDTWESEAZDEECNWIREEEVLNQ

Stronger Transposition

- How would you build a stronger columnar transposition cipher?
- Transpose multiple times** with same or different keywords
 - Myszkowski Transposition** on recurring letters in key

T	O	M	A	T	O
5	3	2	1	6	4
W	E	A	R	E	D
I	S	C	O	V	E
R	E	D	F	L	E
E	A	T	O	N	C
E	null	null	null	null	null

c = ROFOXACDTWESEAZDEECNWIREEEVLNQ

T	O	M	A	T	O
4	3	2	1	4	3
W	E	A	R	E	D
I	S	C	O	V	E
R	E	D	F	L	E
E	A	T	O	N	C
E	null	null	null	null	null

c = ROFOXACDTBEDSEEEACTWWEIVRLENEQ

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Stronger Transposition

- How would you build a stronger columnar transposition cipher?
- **Fractionation:** convert letters into symbols and transpose those
 - E.g., morse code encoding, bits instead of letters
- Suppose **p** = “We are discovered...”
 - **Morse:** 0— 0 02— 0—0 0 —00 00 000 —0—0 ——— 000— 0 0—0 0 —00
 - **Binary:** 01010111 01100101 01100001 01110010 01100101 01100100 01101001 01110011
01100011 01101111 01110110 01100101 01110010 01100101 01100100

Stronger Transposition

- How would you build a stronger columnar transposition cipher?
- **Combine with a substitution cipher**
 - Makes anagram discovery more difficult

6	3	2	4	1	5
W	E	A	R	E	D
I	S	C	O	V	E
R	E	D	F	L	E
E	A	T	O	N	C
E	null	null	null	null	null

$c_1 =$ EVLN**B** ACDT**A** ESEAR**R**
ROFOX**X** DEEC**B** WIREE

Stronger Transposition

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W	E	A	R	E	D
I	S	C	O	V	E
R	E	D	F	L	E
E	A	T	O	N	C
E	null	null	null	null	null

$c_1 =$ EVLN**B** ACDT**A** ESEAR**R**
ROFOX**X** DEEC**B** WIREE

$k_s =$ **ABCAB** **CABCA** **BCABC**

$c_2 =$ EWN**N**C CCEV**A** FUEBT**T**
RPHO**Y** FEFEB**B** XKRFG



Questions?



Cipher Metrics

Confusion and Diffusion

■ “Confusion”

- Every bit of the ciphertext should depend on **several parts** of the plaintext
- Maintains that the ciphertext is statistically independent of **the plaintext**

■ “Diffusion”

- A change to one plaintext bit should change **50%** of the ciphertext bits
- A change to one ciphertext should change **50%** of the plaintext bits
- Plaintext features **spread** throughout the entire ciphertext

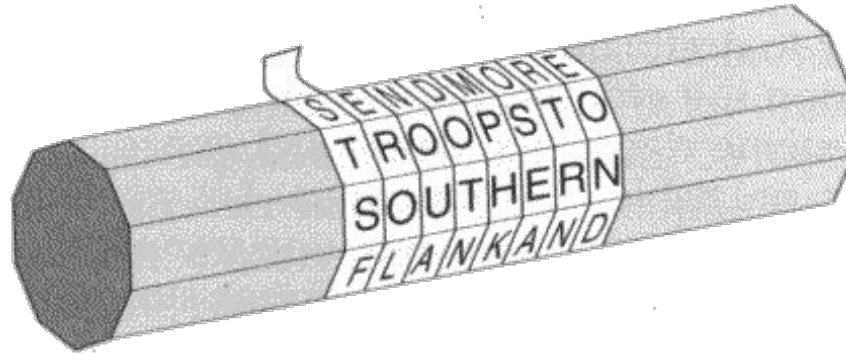
- These are **cipher metrics**—how we “weigh” a cipher’s security

Cipher Metrics: Transposition Ciphers

- Do **transposition ciphers** achieve confusion or diffusion?

Cipher Metrics: Transposition Ciphers

- Do **transposition ciphers** achieve confusion or diffusion?
 - **Diffusion**—they spread the plaintext around!

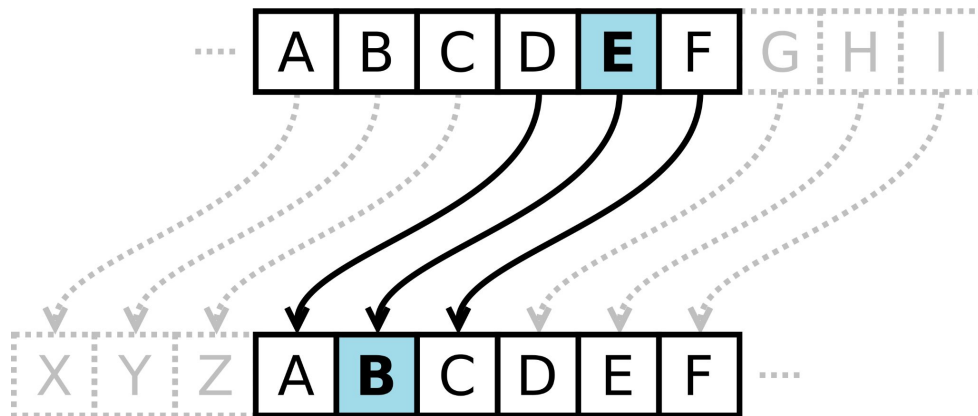


Cipher Metrics: Substitution Ciphers

- What level of confusion & diffusion do ***simple substitution ciphers*** have?

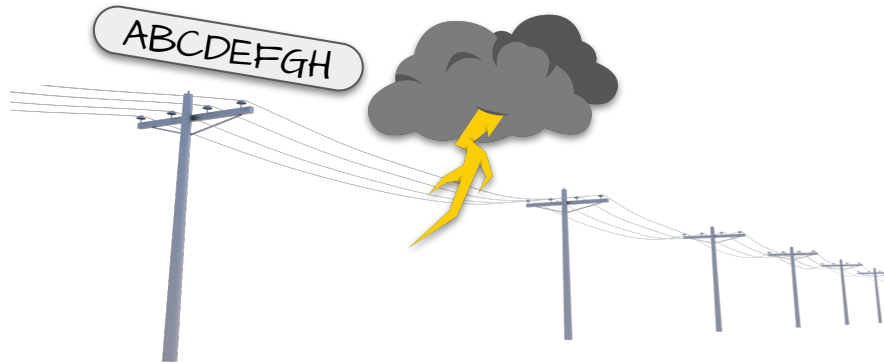
Cipher Metrics: Substitution Ciphers

- What level of confusion & diffusion do **simple substitution ciphers** have?
 - **None**—hence why frequency analysis is useful
 - Changing one plaintext or key symbol changes one ciphertext symbol



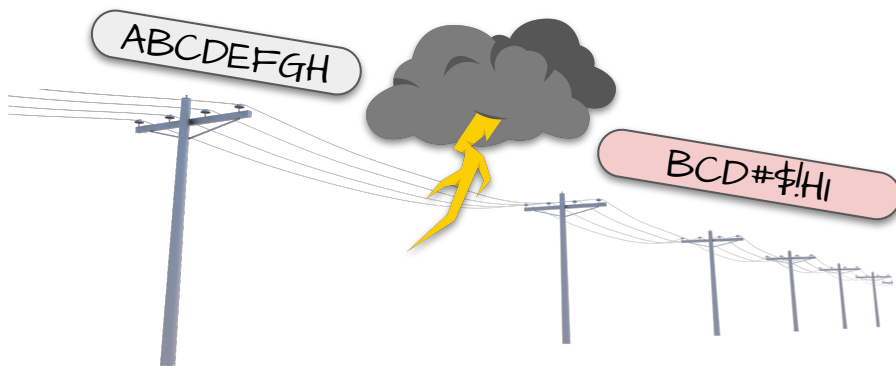
Cipher Metrics: Noisy Channels

- How does **low diffusion** impact communication across a **noisy channel**?



Cipher Metrics: Noisy Channels

- How does **low diffusion** impact communication across a **noisy channel**?
 - Low diffusion = **more tolerant** to corrupted symbols



Questions?



Next time on CS 4440...

Block ciphers, AES, secure channels